# Constitutive and machine learning models of compaction rate based on brachistochrone and environmental considerations

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**Abstract**. In geo-environmental engineering, compaction is an important mechanical operation, which improves the strength behavior of construction materials. This is achieved by applying rammers of various energies; in the case of the field and a mould and mass of hammer (m), which drops freely from a height (H); in the case of a laboratory exercise to deliver compaction energy (E) to the sub-grade soil. However, the field or laboratory time is enormous, and attempts are made to decrease the time spent during these procedures prior to geo-structural designs. One of those attempts is either to optimize the time-speed technique. This implies the minimization of time and maximization of the speed of hammer drop through the height (H). To fully compact soil to attain desired density for safe earth and concrete works, for instance, in flexible and rigid pavements, a number of blows or passes are needed by design standards for each field or laboratory exercise, which requires time. So, in this research work, intelligent models have been proposed on the considerations of the Brachistochrone problem domain solved by the Euler-Lagrange approach. This was formulated for the time (t) taken for a compaction rammer to pass or hammer to drop from H under gravity (g) and zero friction to deliver compaction energy to a compacted soil mass. Initially, the path equation of Brachistochrone for time (t) was formulated and solved to create the parametric equations upon which the time (t) is minimized. Lastly, a numerical database was generated from the analytical solution based on Brachistochrone and deployed into the learning abilities of genetic programming (GP), artificial neural network (ANN), and evolutionary polynomial regression (EPR).

## 1. Introduction

The problem of Brachistochrone shows the calculus of variation-based formulated principle for the solution of constitutive models, which try to sufficiently solve geo-environmental and earthwork problems, which are influenced by the force of gravitation [1]. This has been identified as constitutive problems between points on a path considered the path of fastest distance [1-3]. Various calculi of variation methods are being deployed to solve the very many geo-environmental and geotechnics extremum situations to deal with points of stationary in order to optimize variational solutions [2].

In very simple indices, a calculus of fundamentals of 1-dimensional state can be correlated with calculus of variation, i.e., a function y = f(x) understanding in a selected variable, for the domain x ϵ R [3]. Assume that the y = f(x) function is a differentiable and continuous factor in the domain of consideration, considered to be x ϵ R, then the extrema; local and global can be solved to estimate the required function. This expectedly happens at some $x_i$ ϵ R domain by observing the initial two derivatives. Likewise, the calculus of variation is the close examination of the functional shown in Equation 1:

$$E[y] = \int_a^b F\left(t, y(t), y'(t)\right) dt \tag{1}$$

where the integrand $F\left(t, y(t), y'(t)\right)$ is a function of the dependent parameter t, a function y(t), and $y'(t)$, which is the initial derivative with the differentiable notation symbolizing the differential associated to t [3]. An Euler-Lagrange-based curvature-dependent equation can be mathematically estimated from Equation (1) to evaluate the distance of the shortest path between two locations by deploying the constitutive abilities of the principles of Hamilton [1].

Relating the Brachistochrone solution to the geo-environmental and geotechnical engineering problems to earthworks, for example, settlement, California bearing ratio, unconfined compressive strength, consolidation, erodibility, geophysical flows, stability of slopes, etc., these are experimentally studied and evaluated by mechanically enhancing their compactness or the mass of soil's ability to attain optimal density [4-10]. To achieve this, the proctor is used or the West African standard methods of different compacting energies delivering energies in kJ/m$^3$ (kN.m/m$^3$) dropping from point (H) influenced by gravitational field [1, 3-5, 9]. The method of the hammer drop is ree drop by design standard and impacts through a fixed drop distance (H), fixed speed of fall, (v), and frictionless. The optimized time (t) required to achieve compaction has been derived by using the Brachistochrone solution in Equation 2 as follows according to Onyelowe and Ebid [3]:

$$t\big(a(\theta - sin\theta)\big) = \sqrt{\frac{1}{2ga}}\left(\frac{1}{\sqrt{2}}\left[\left(Ln\left(\sec\left(\frac{\theta}{2}\right) - 1\right) - Ln(\sec\left(\frac{\theta}{2}\right) + 1)\right)\right] + \sqrt{2}\left[Ln\left(\sec\left(\frac{\theta}{2}\right) + \tan\left(\frac{\theta}{2}\right)\right)\right]\right) \tag{2}$$

where, θ represents the values within the allowable domain a-b between 0 and π/2.

## 2. Optimized representation of the time (t) of compaction blow

### *2.1 Analytical and numerical representation*

In Equation 2, the Brachistochrone equation is presented for compacting earth-structure setup, in which the time (t) used for a compacting hammer of mass (m) to drop under gravitational field (g) from a height (H) is optimized. This analytical method has been graphically represented in Figure 1(a). This shows that the Brachistochrone solution for the analytical problem for soil mechanical stabilization at an efficiency of 99.33% has been achieved and with a supported model performance by [6]. Meanwhile, the θ data within the allowable problem domain ($0 < \theta < \frac{\pi}{2}$) were further deployed to achieve a Brachistochrone-based solution of numerical expression as presented in Figure 1(b). It further shows that the numerical and analytical solutions compared well in terms of performance efficiency with 99.31% accuracy, which shows a difference of 0.02%. With over 99% accuracy of prediction for the compaction time based on Brachistochrone models at optimized time, (t) indicates that with velocity (v) at optimal, the pressure of impact on the soil mass being mechanically stabilized will at the same time be optimized, which eventually will optimize the compacting pressure [8, 9]. The present innovative result is proposed to be deployed in the fabrication and calibration of a motorized and laboratory modified compaction machine with the capability to deliver blows at an optimized velocity (v)-

optimized rate (t). This will eventually provide an optimized amount of compacting pressure. Figures 1(c) and 1(d) are the curves of the cycloid of the real Brachistochrone for the respective theoretical and mathematical model solutions. These show the real curve path of the hammer as it drops from point H [7]. The respective accuracies of models evaluation for the optimal time (t) curve for the theoretical and mathematical model solutions are 97% and 97.6%, which agrees with [6].

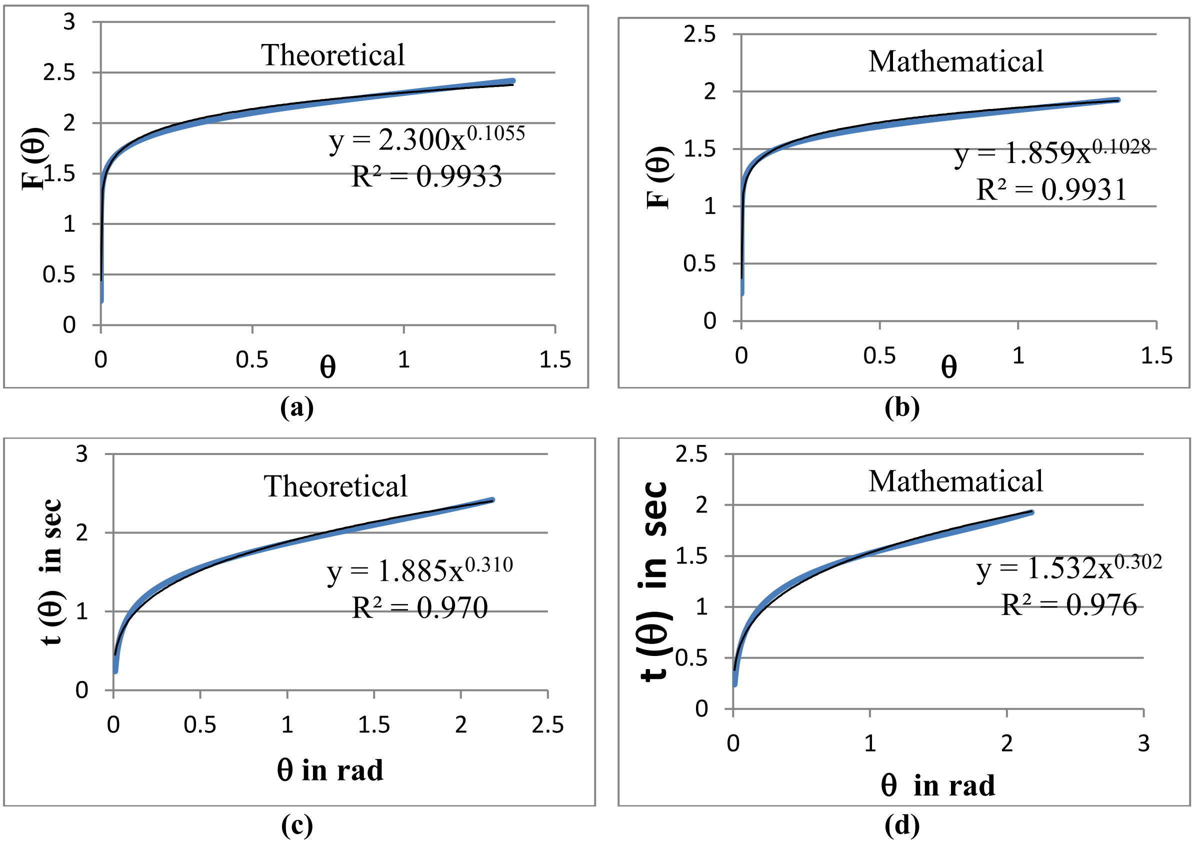



**Figure 1.** Illustration of the mathematical models for the optimized time (t) versus the expression parameters of Brachistochrone.

*2.2 Soft computing representation based on brachistochrone considerations*

The database of 218 values with respect to minimized time (t) generated through analytical and numerical considerations of variational calculus of Brachistochrone comprising θ, y, and H as the input variables was intelligently analyzed further to predict intelligent models for the optimized time (t). First, a Pearson correlation test in Table 1, which shows a correlation relationship of above 85% [11] between t and θ, H, and a histogram in Figure 2 showing the distribution of the database for the input and output were conducted on the data, are presented.

**Table 1.** Pearson correlation matrix

| | **θ** | **y** | **H** | **t** |
|---|---|---|---|---|
| **θ** | 1.000 | | | |
| **y** | 0.932 | 1.000 | | |
| **H** | 0.987 | 0.972 | 1.000 | |
| **t** | 0.938 | 0.786 | 0.877 | 1.000 |

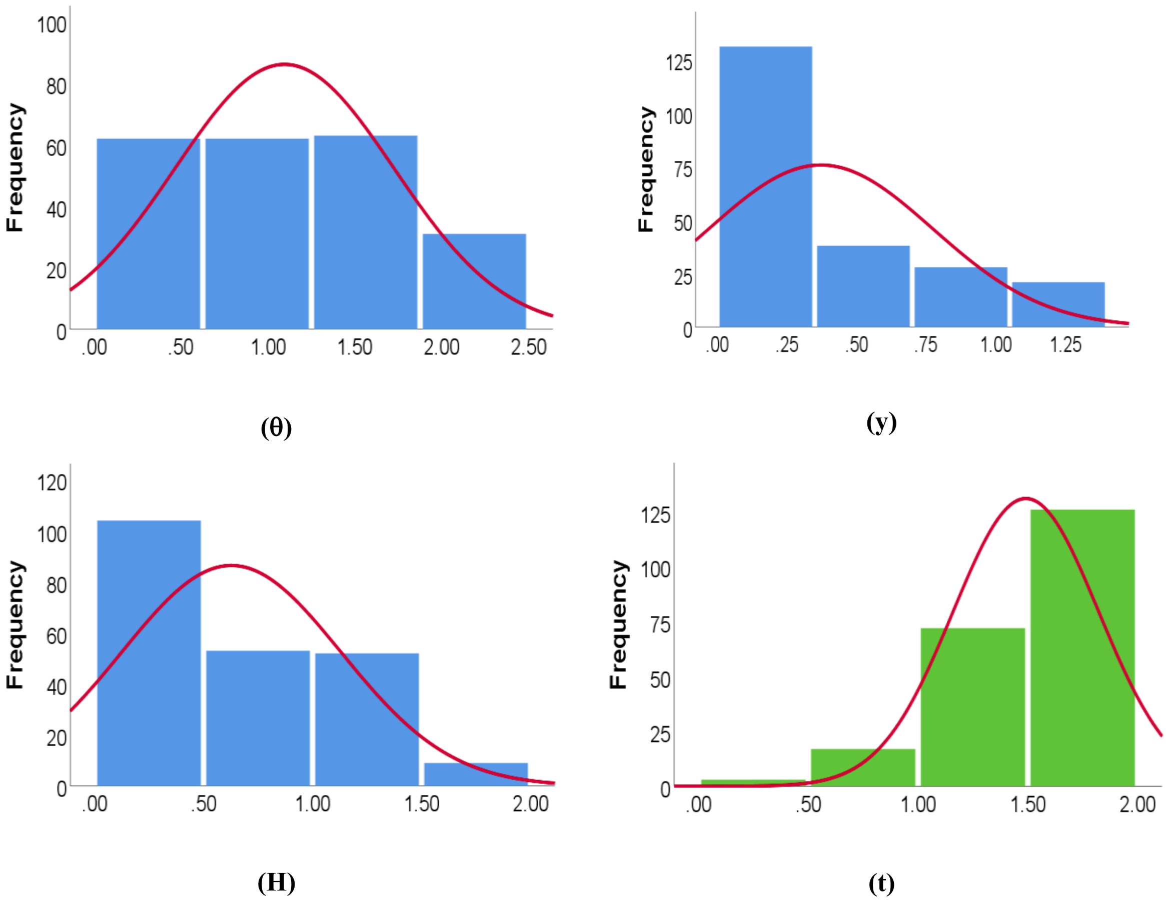



**Figure 2.** The independent variables (in blue) and the dependent variable (in green) distribution histogram.

Different intelligent methods [9, 11] were applied to propose the minimum time (t) needed for a compaction hammer to fall by Brachistochrone path of shortest time over a distance; height (H) of compaction hammer. These intelligent procedures are genetic programming (GP), artificial neural network (ANN), and polynomial linear regression enhanced by using a genetic algorithm, which is known as evolutionary polynomial regression (EPR). All the proposed intelligent models were deployed to predict the values of (t) using (θ), (y), and (H) as independent variables. Each model was based on a different approach, which included the evolutionary approach for GP, ANN's process of mimicking biological neurons, and EPR's enhanced mathematical regression procedure. Meanwhile, for all developed models, the model performance accuracy was estimated in terms of the sum of squared errors (SSE). The developed GP model has only 3 levels of complexity. The population size, the survivor size and the number of generations were 10000, 3000 and 100, respectively. Equation 3 presents the output formulas for (t). The average error % of the total set is (1.7%), while the ($R^2$) value is (0.994).

$$t = 1.53\ \theta^{0.277} \tag{3}$$

A back-propagation trained (ANN) with one hidden layer and hyperbolic tangent (Hyper-Tanh) activation function was used to predict the same (t) values. The used network layout and its connection weights and the values for the degrees of influence for each selected variable in the model, which indicated that (θ) is the most important factor, then (H) and (y), are illustrated in Figure 3. The average error % of the total dataset for this network is 0.3%, and the $R^2$ value is 1.000.

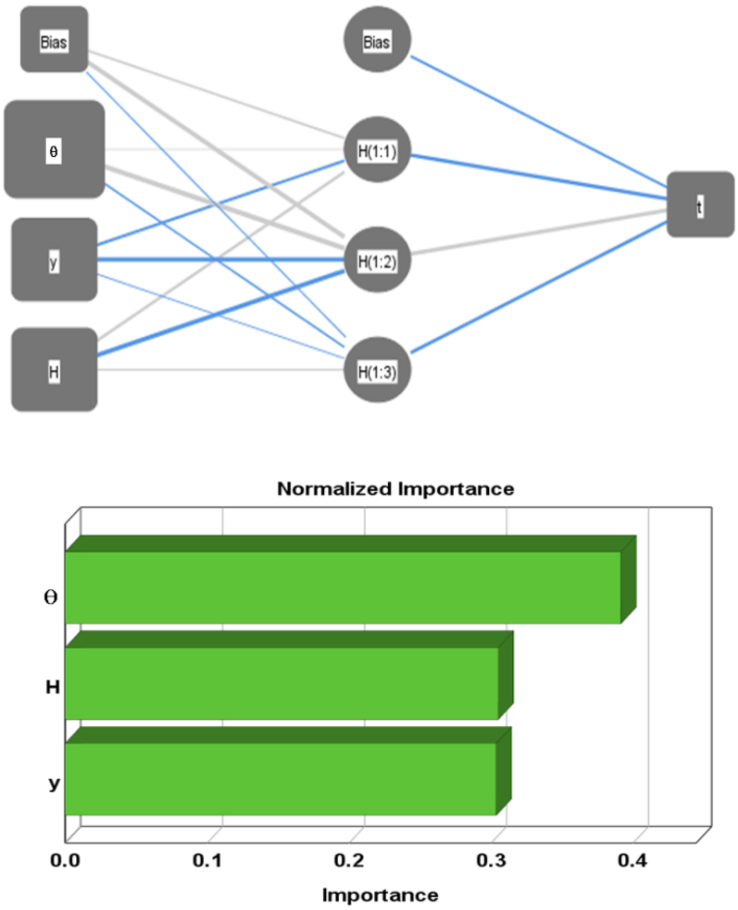



**Figure 3.** Architecture layout for the developed ANN and its connection weights and the relative importance of input parameters.

Finally, the developed EPR model was limited to quadratic level, for 3 inputs; there are 10 possible terms (6+3+1=10) as follows in Equation 4:

$$\sum_{i=1}^{i=3} \sum_{j=1}^{j=3} X_i \,.\, X_j + \sum_{i=1}^{i=3} X_i + C \qquad (4)$$

A genetic algorithm intelligent procedure was applied on the ten indexes from which the most effective 6 indexes were selected to propose the values of (t). The outputs are illustrated in Equation 5, and the fitness and performance accuracies of the three models are shown in Figure 4 and Table 2. The average error in % and $R^2$ values were 0.5% and 1.000 for the total datasets. Lastly, the Taylor diagram showing the predicted models' correlation coefficient and the variances showing the variance between the analytical/numerical and predicted data are presented in Figure 5. The closed-form equation can be applied at maximum performance to predict the time of the shortest path to deliver compaction energy to a mechanically stabilized soil. This is further done by considering the optimal performance of the independent variables.

$$t \;=\; 1.04 + \; 4.37\frac{\theta}{H} - 0.24\,\theta^2 - \frac{8.78}{\theta} - \frac{0.63}{1000000y} + \frac{0.333}{1000H} \qquad (5)$$

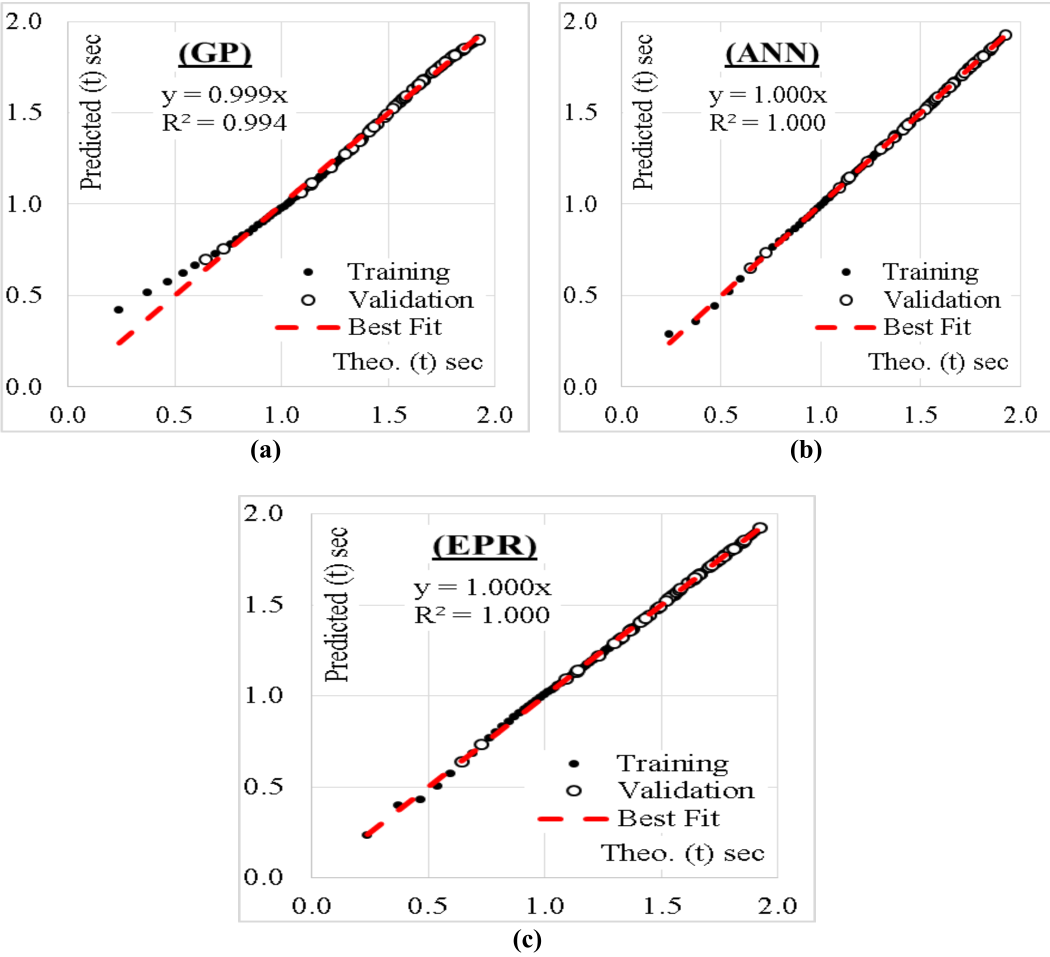
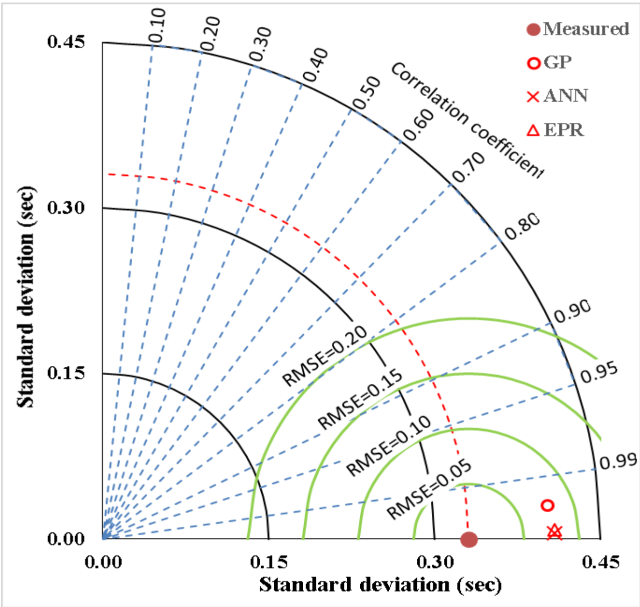


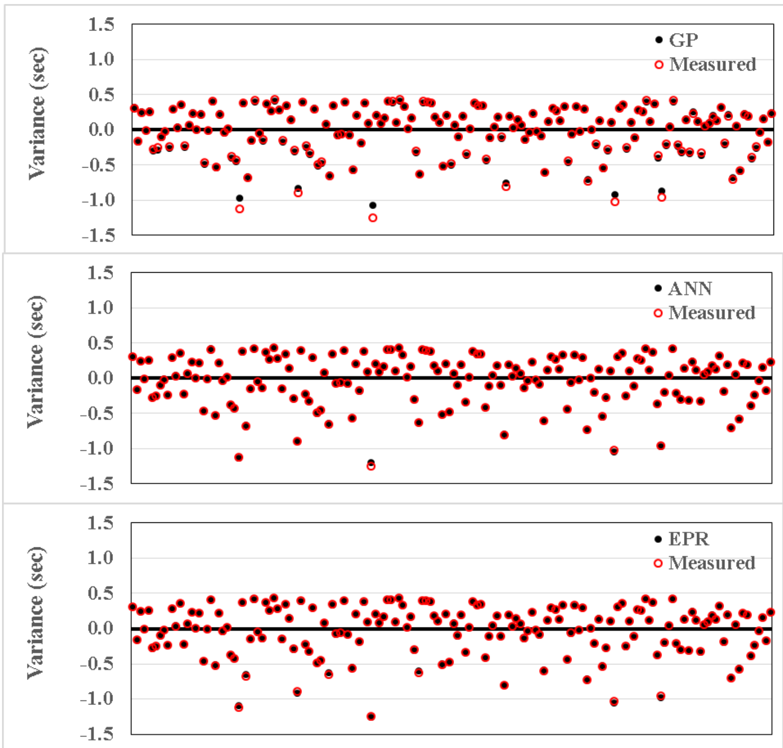

**Figure 4.** Relation between the predicted and the theoretical (t) values utilizing the developed models.

**Table 2.** Accuracies of performance of the developed models

| Technique | Developed model | SSE | Error % | $R^2$ |
|---|---|---|---|---|
| GP | Equation (3) | 0.139 | 1.7 | 0.994 |
| ANN | Figure (3) | 0.004 | 0.3 | 1.000 |
| EPR | Equation (5) | 0.010 | 0.5 | 1.000 |




**(a)**




**(b)**

**Figure 5.** Representation of the correlation coefficient, standard deviation, and root mean square error (RMSE) in (a) Taylor diagram and the variances (b) between analytical/numerical and predicted values of t.

## 3. Conclusions

This research paper presents intelligent models, which has used genetic programming, artificial neural network, and evolutionary polynomial regression to propose the values of minimum time (t) using (θ), (y), and (H) as predictors. From the foregone predictive model exercises, the following conclusions have been remarked:

- Both artificial neural network (ANN) and the evolutionary polynomial regression (EPR) have almost the same prediction performance accuracy; 100% with an error of 0.3% and 100% with an error of 0.5%, respectively, while the genetic programming (GP) model with the indices of performance as 99.4% and 1.7% for the $R^2$ and the average error, has the lowest accuracy but still above 85% allowable for designs.
- However, the prediction residual in percent of the artificial neural network and the evolutionary polynomial models may be near; the outcome of the EPR model is a closed form proposed equation, which is equally applied manually as well as in software applications unlike the ANN outcome, which cannot be manually applied.
- The weights of absolute summation of the neurons in the layer of independent variables of the ANN model show that (θ) had major influences on (t), than (y) and (H).
- The ten indices of the usual quadratic formula of the polynomial regression were successfully reduced to six indexes by the genetic algorithm procedure without any serious impact on the models' accuracy.
- Finally, it is important to verify the prediction accuracy of the models when utilized beyond the range of this present prediction.

## References


[1] Onyelowe K C (2021) Application of Calculus of Variation in the Optimization of Functional Parameters of Compacted Modified Soils: A Simplified Computational Review. *Mathematical Problems in Engineering*, **2021**, 6696392, 1–6, https://doi.org/10.1155/2021/6696392.

[2] Deshmukh P C, Rajauria P, Rajans A, Vyshakh B R and Dutta S (2017) The Brachistochrone. *Resonance*, **22**, 847–866, https://doi.org/10.1007/s12045-017-0539-1.

[3] Onyelowe K C and Ebid A M (2022) Compaction energy-hammer fall rate (E-t) constitutive model by Brachistochrone technique. *Computational and Applied Mathematics*, **41**(5):224, 1–14, https://doi.org/10.1007/s40314-022-01937-9.

[4] Bowles J E (1997). Foundation Analysis and Design; International Edition. The McGraw-Hill Companies, Inc., New York.

[5] Bowles J E (1986) Engineering properties of soils and their measurement; 3rd Ed. McGraw-Hill, New York.

[6] Onyelowe K C, Kontoni D-P N, Ebid A M, Dabbaghi F, Soleymani A, Jahangir H and Nehdi M L (2022) Multi-Objective Optimization of Sustainable Concrete Containing Fly Ash Based on Environmental and Mechanical Considerations. *Buildings*, **12**(7):948, 1–25, https://doi.org/10.3390/buildings12070948.

[7] Stutz H and Mašín D (2017) Hypoplastic interface models for fine-grained soils. *International Journal for Numerical and Analytical Methods in Geomechanics* **41**(2), 284–303, https://doi.org/10.1002/nag.2561.

[8] Insana A, Beroya-Eitner M A, Barla M, Zachert H, Žlender B, van Marle M, Kalsnes B, Bračko T, Pereira C, Prodan I, Szymkiewicz F and Löfroth H (2021) Climate Change Adaptation of Geo-Structures in Europe: Emerging Issues and Future Steps. *Geosciences*, **11,** 488, https://doi.org/10.3390/geosciences11120488.

[9] Ebid A M, Onyelowe K C and Salah M (2022) Estimation of bearing capacity of strip footing rested on bilayered soil profile using FEM-AI-coupled techniques. *Advances in Civil Engineering*, **2022**:8047559, 1–11, https://doi.org/10.1155/2022/8047559.

[10] Vogelsang J, Zachert H, Huber G and Triantafyllidis T (2015) Effects of Soil Deposition on the Initial Stress State in Model Tests: Experimental Results and FE Simulation. *In: Triantafyllidis, T. (eds) Holistic Simulation of Geotechnical Installation Processes. Lecture Notes in Applied and Computational Mechanics, vol 77, pp. 1–20. Springer, Cham. https://doi.org/10.1007/978-3-319-18170-7_1.*

[11] Onyelowe K C, Ebid A M, Nwobia L (2021) Evolutionary Prediction of Soil Loss from Observed Rainstorm Parameters in an Erosion Watershed Using Genetic Programming. *Applied and Environmental Soil Science*, **2021**:2630123, 1-15, https://doi.org/10.1155/2021/2630123.